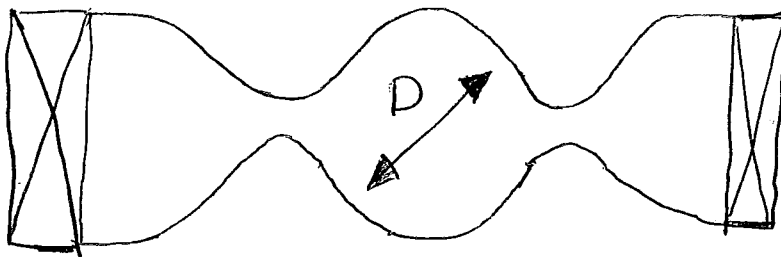


## Exam Mesoscopic Physics 6-7-2010

Indicate for every answer how it is obtained. There are 5 questions.  
Please use separate sheets for each question. Write name and student number on each sheet

1) Consider the following structure below. It consists of a cavity (with diameter  $D$ ) made in a two-dimensional electron gas which is connected to two reservoirs via two constrictions with width  $W$ . The transport is ballistic and the reflection from the sample boundaries is specular.



- Draw a few typical trajectories for electrons that are transmitted through the structure
- Draw a few typical trajectories for electrons which are reflected

The width of each constriction is chosen to be 10 times the Fermi wavelength. Therefore each accommodates 10 quantum channels

- Give an approximate estimate of the electrical conductance of the structure.

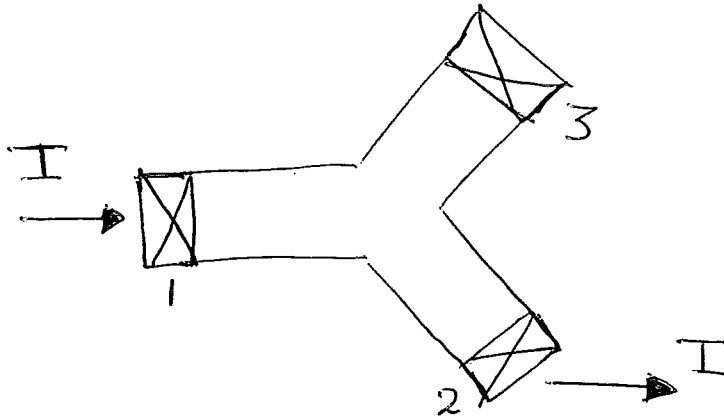
Now both constrictions are made narrower, in such a way that there is only one quantum channel left in each constriction.

- Indicate how that changes the conductance.

Now consider a similar structure where the transport is diffusive and the electron mean free path is much shorter than the size  $D$  of the cavity. At low temperatures the conductance is measured as a function of applied perpendicular magnetic field.

- Describe how the conductance depends on magnetic field. Indicate what processes can be important, and in what range of magnetic field they can be observed.

2) Consider a the following ballistic system . It has 3 contacts, which support  $N$  quantum channels. Consider the transport as classical (particle like)



a) Give the Landauer Buttiker equations for the currents/voltages in all contacts (You don't have to solve the equations)

b) Give an estimate for magnitude of the transmission and reflection coefficients.

A perpendicular magnetic field is applied, which will deflect electrons to the right.

c) Describe (qualitatively) how this will change the reflection and transmission coefficients.

d) When the magnetic field exceeds a certain value, some transmission and reflection coefficients have become zero. Which are these and why is this?

3) Consider electrons in a box width length  $L$ , width  $W$  and thickness  $D$ . The Fermi energie is  $E_f$ , and the effective mass is  $m^*$ .

Start with a situation where  $L$ ,  $W$  and  $D$  are large so that the density of states is 3 dimensional

a) Now the thickness  $D$  is reduced. ( the Fermi energy is kept constant). At which thickness has the system become 2-dimensional?

b) Keeking this thickness, the width  $W$  is reduced. At which width the system has become 1 dimensional?

c) Finally the length is reduced too. At which length has the system become zero dimensional?

Explain your answers!

- 4) Consider a superconducting Josephson junction
- Give the so called Josephson equations for the (super) current through the junction and the voltage across it.
  - One of these relations is exact, the other one is a (good) approximation. Which is exact, and which one is a (good) approximation?
  - Give the (differential) equations for the current through the junction which also includes the possible (Ohmic) tunnel resistance and the effect of the junction capacitance
  - Do you consider the dynamics which is described with the above equation as “classical” or “quantum”. Clarify your answer!

The capacitance of the junction is now reduced.

- What are the conditions so that (macroscopic) quantum effects can be observed?
- Describe a circuit which can be used to study these macroscopic quantum effects.

5)

- Draw a two dimensional system which can be used for quantum Hall measurements. Also indicate the source drain contacts and the contacts for measurement of the longitudinal and Hall resistances.
- Illustrate the edge channel flow for the case of two occupied Landau levels
- Indicate and describe under what conditions the longitudinal resistance is zero and the Hall resistance is quantized.

Because under “quantum Hall “conditions the longitudinal resistance becomes zero, the transport in this case is sometimes referred to as “dissipationless”

- Is this correct and is indeed the transport dissipationless? If not, indicate where the dissipation takes place.

In an experiment the Fermi energy is tuned so that the quantum Hall effect is observed with two occupied Landau levels. Now the current from source to drain contact is increased and it is observed that the quantum Hall effect breaks down, and a finite longitudinal voltage is observed.

- Give some possible mechanisms why the quantum Hall effect breaks down at high currents.

